Nambu-Goto string with massive ends at finite temperature

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Abstract

It is shown that the critical (or deconfinement) temperature for the Nambu-Goto string connecting the point-like masses (quarks) does not depend on the value of these masses. This temperature turns out to be the same as that in the case of string with fixed ends (infinitely heavy immobile quarks).

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1 Introduction

In papers [1,2] the dependence of the interquark potential on the quark masses has been investigated in the model of the relativistic string with ends loaded by point-like masses. It was shown that allowance for the finite values of quark masses leads to considerable corrections to string potential in comparison with calculations when the string ends are fixed (immobile quarks with infinitely heavy masses). The critical radius[1] in the string potential turns out to be dependent on the quark masses especially in the case of the asymmetric configurations when string ties together light and heavy quarks [2].

Along with the critical radius of the interquark potential, an important parameter of the string model of hadrons is the critical or deconfinement temperature [4]. This prediction of the string model is directly compared with the lattice simulations in the framework of the gauge theories [5]. In this connection the investigation of the quark mass contribution to the critical string temperature is of certain interest. It is this problem that will be treated in this paper.

Usually one believes that in string models the critical radius of the potential R_c and the critical temperature are related. For example, in the Nambu-Goto string with fixed ends the relation $2\,R_c T_c = 1$ holds. As the critical radius R_c depends on the quark masses [1], one could expect that this will be valid for the critical temperature T_c also. However this is not so. The deconfinement temperature in the Nambu-Goto string with point-like masses (quarks) at its ends turns out to be independent on the quark masses, this temperature being the same as that in the string with fixed ends.

The layout of the paper is the following. In Sec. II the definition of the critical temperature in string models is reminded in short. In Sec. III a new approach to calculation of the Casimir energy at finite temperature in the Nambu-Goto string of finite length is suggested. The temperature dependence of this energy determines the deconfinement or critical temperature in the string model under consideration. In Sec. IV it is shown that the critical temperature in the Nambu-Goto string model with fixed ends and with point-like masses attached to string ends is the same. In Conclusions (Sec. V) the obtained results are discussed in short.

2 Critical temperature in string models

The critical temperature (or the temperature of deconfinement) in string models is determined in the following way. If $V(R,\,T)$ is the string potential calculated at finite temperature T then its asymptotics at large distances is

$$V(R,\,T) \to \sigma(T)\cdot R, \quad R\to\infty, \tag{2.1}$$

[1]As known [3] the potential generated by string is not determined for any distances between quarks no sooner than at $R > R_c$, where R_c is the critical radius in the Nambu-Goto string with fixed ends which is determined by the string tension M_0^2: $R_c = \pi\,(D-2)/(12\,M_0^2)$.

where $\sigma(T)$ is an effective string tension depending on the temperature T. At $T = T_c$ the string tension vanishes

$$\sigma(T_c) = \lim_{R\to\infty} R^{-1} V(R, T_c) = 0\,. \tag{2.2}$$

The string potential at finite temperature is calculated directly in the same way as at $T = 0$ [1]. As a result one obtains the well known square-root expression

$$V(R, T, m) = M_0^2 R \sqrt{1 + \frac{2(D-2)}{M_0^2 R}\, E_C(R, T, m)}\,, \tag{2.3}$$

where M_0^2 is the string tension at zero temperature $\sigma(T = 0) = M_0^2$, m is the quark mass, $E_C(R,\, T,\, m)$ is the renormalized Casimir energy for one transverse degrees of freedom in string model, and D is the dimension of the space-time. Usually one puts $D = 4$. Hence, determination of the critical temperature requires calculation of the Casimir energy at finite temperature in the string model under investigation.

3 Casimir energy at finite temperature

The Casimir energy in the Nambu-Goto string at finite temperature is given by the following formula [1]

$$E_C(R, T) = \frac{T}{2} \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{\infty} \ln(\Omega_n^2 + \omega_k^2)\,, \tag{3.1}$$

where Ω_n are the Matsubara frequencies $\Omega_n = 2\pi n T, \quad n = 0, \pm 1, \ldots$; T is the temperature, and ω_k are the eigenfrequencies of the string determined by the boundary conditions for the string coordinates. If the string ends are fixed (immobile quarks) then

$$\omega_k = \frac{k\pi}{R}, \quad k = 1, 2, \ldots\,. \tag{3.2}$$

In the case of the string with masses at its ends the frequencies ω_k are the positive roots of the following equation [1,2]

$$\tan(\omega\, R) = \frac{2\omega m}{\omega^2 - M_0^2}\,. \tag{3.3}$$

For simplicity the symmetric quark configuration is considered $m_1 = m_2 = m$. Keeping in mind that the critical temperature is determined by the value of $E_C(R, T)$ at $R \to \infty$ (see Eqs. (2.2) and (2.3)), it is convenient, in the case of string frequencies (3.1), to do this limiting transition directly in Eq. (3.1) substituting the summation over k by integration

$$E_C^{\text{fixed}}(R \to \infty, T) = \frac{T}{4} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dk \ln\left[\Omega_n^2 + \left(\frac{k\pi}{R}\right)^2\right]\,. \tag{3.4}$$

Analytical regularization gives the following finite value of the divergent integral over dk

$$\int\limits_{-\infty}^{+\infty} \frac{dk}{2\pi} \ln(a^2 + k^2) = a\,. \tag{3.5}$$

Taking account of this one obtains from Eq. (3.4)

$$E_C^{\text{fixed}}(R \to \infty, T) = \frac{RT}{2} \sum_{n=-\infty}^{+\infty} \Omega_n = 2\pi R T^2 \sum_{n=1}^{\infty} n = 2\pi R T^2 \zeta(-1) = -\frac{\pi R T^2}{6}\,, \tag{3.6}$$

where $\zeta(z)$ is the Riemann zeta function. Substituting (3.6) in (2.3) we find the critical temperature in the Nambu-Goto string with fixed ends

$$\frac{T_c}{M_0} = \sqrt{\frac{3}{\pi(D-2)}}\,. \tag{3.7}$$

However this method cannot be applied to string frequencies determined by Eq. (3.3). Investigation of the double sum in (3.2) without introducing the integration over dk (see [6]) is again based on the fact that frequencies ω_k are multiples of π/R.

Here we suggest a new method for calculating the Casimir energy at finite temperature that equally well works both with frequencies (3.2) and with string frequencies determined by Eq. (3.3). The idea of the method is the following. At first we represent the renormalized Casimir energy at zero temperature in terms of the integral over string eigenfrequencies. In other words, we obtain the spectral representation for $E_C(R,T)$

$$E_C(R, T=0) = \int\limits_0^{\infty} d\omega \mathcal{E}_C(R,\omega)\,, \tag{3.8}$$

and then pass in a standard way [7,8] from integration to summation over the Matsubara frequencies $\Omega_n = 2\pi nT, \quad n = 0, \pm 1, \pm 2, \ldots$. Practically this can be done by the following substitution in (3.8)

$$d\omega \to 2\pi T\, \delta(\omega - \Omega_n)\,, \tag{3.9}$$

with result

$$E_C(R,T) = 2\pi T \sum_{n=0}^{n=\infty}{}' \mathcal{E}_C(R, \Omega_n)\,. \tag{3.10}$$

The prime in sum sign means that the term with $n = 0$ should be taken with multiplier $1/2$.

In our preceding paper [1] we derived the following integral representation for the Casimir energy in the Nambu-Goto string with fixed ends

$$E_C^{\mathit{fixed}} = \frac{1}{2\pi} \int\limits_0^{\infty} d\omega\, \ln\left(1 - \exp\left(-2\omega R\right)\right) = -\frac{R}{\pi} \int\limits_0^{\infty} \frac{\omega\, d\omega}{\exp\left(2\omega R\right) - 1}\,. \tag{3.11}$$

The last expression in this formula is obtained by integration by parts. Omitting the sign minus in (3.11) we see that the spectral density of energy in this formula is of plankian

form for the one-dimensional black body, the effective temperature of the string vacuum being equal to $(2R)^{-1}$. Applying the algorithm explained above we find

$$E_C^{\rm fixed}(R,T) = -4\pi T^2 R \sum_{n=0}^{\infty}{}' \frac{n}{\exp{(4\pi nRT)}-1}\,. \tag{3.12}$$

Integration by parts in (3.11) was required for obtaining the term with $n=0$ in the sum (3.12) without divergencies. In analogous calculations in statistical physics [7,8] for overcoming the stated difficulty the Casimir force is calculated at first and then on this basis the corresponding potential is recovered.

The sum in (3.12) can be evaluated in two limiting cases, for large and for small temperatures. At large T the main contribution in (3.12) is due to the term with $n=0$

$$E_C^{\rm fixed}(R, T\to\infty) = -\frac{T}{2}\,. \tag{3.13}$$

At small T the Euler-Maclaurin formula

$$\sum_{n=0}^{\infty}{}' f(n) = \int\limits_0^{\infty} f(x)\,dx - \frac{1}{12}f'(0) \tag{3.14}$$

can be used. In the case under consideration

$$f(x) = \frac{x}{\exp{(4\pi TRx)}-1} \quad \text{and} \quad f'(0) = -\frac{1}{2}\,. \tag{3.15}$$

As a result, we obtain for small T

$$E_C^{\rm fixed}(R,T) = -\frac{\pi}{24\,R} - \frac{\pi T^2 R}{6}\,. \tag{3.16}$$

Proceeding from the physical consideration, it is clear that the string picture of quark confinement inside the hadrons is applicable only at low temperatures. In string models the temperature scale is determined by the string tension $M_0 \sim 0.4$ GeV. Hence under finding the critical temperature (or the deconfinement temperature) in string models, the region of small temperatures should be considered [4]. With allowance for all this, we have to substitute in (2.1) and (2.3) the expression for the Casimir energy at small T (Eq. (3.16)). After taking the limit $R\to\infty$ in (2.2) contribution of the first term in (3.16) to the effective string tension vanishes and we obtain the critical temperature (3.7).

By making use of the formula (3.16) one can formally introduce a critical temperature for string of a finite length $\bar{T}_c(R)$. To this end, $\lim_{R\to\infty}$ should be removed from the definition (2.2). At this temperature, the effective tension of the string of finite length R vanishes. It is obvious that the string critical temperature defined in this way will be dependent on the string length R. By making use of (3.16), (2.2), and (2.3) we obtain

$$\bar{\tau}_c^2(\rho) = \tau_c^2 - \frac{1}{4\rho^2}\,, \tag{3.17}$$

where the following dimensionless variables are introduced

$$\tau_c = T_c/M_0, \quad \rho = M_0 R\,. \tag{3.18}$$

It is clear that when $\rho \to \infty$ the critical temperature $\bar{\tau}_c(\rho)$ dependent on the string length ρ tends to its limiting value τ_c from below because the existence of longer strings requires a greater work for their splitting (phase transition) than it takes place in the case of short strings.

In the framework of other approach, the critical temperature for the Nambu-Goto string of finite length has been considered in paper [9]. The results obtained there probably imply that at $\rho \to \infty$ $\tau_c(\rho)$ tends to τ_c from below.

4 Critical temperature for the Nambu-Goto string with massive ends

The method for calculating the Casimir energy at finite temperature presented in preceding section can be directly applied to the Nambu-Goto string with massive ends. In our preceding paper [1] the following integral representation for the Casimir energy at zero temperature in this string model has been derived

$$E_C(R, T=0, m) = \frac{1}{2\pi}\int\limits_0^\infty d\omega \ln\left[1 - e^{-2\omega R}\left(\frac{\omega m - M_0^2}{\omega m + M_0^2}\right)^2\right]\,, \tag{4.1}$$

where m is the quark mass. For simplicity, the quarks with equal masses are considered. The Casimir energy and the string potential for different quark masses $m_1 \neq m_2$ have been considered in [2].

Integrating by parts in (4.1) one obtains

$$E_C(R, T=0, m) == -\frac{R}{\pi}\int\limits_0^\infty \frac{\omega\, d\omega}{g(\omega)-1}\left(1 - \frac{2mM_0^2}{R(\omega^2 m^2 - M_0^4)}\right)\,, \tag{4.2}$$

where the notation

$$g(\omega) = e^{2\omega R}\left(\frac{\omega m + M_0^2}{\omega m - M_0^2}\right)^2 \tag{4.3}$$

is introduced for smothering. It is interesting to note that the spectral density of energy in (4.2) is not plankian now.

Along similar lines we arrive at the Casimir energy at finite temperature

$$E_C(R, T, m) = -4\pi T^2 R \sum_{n=0}^{n=\infty}{}' \frac{n}{g(2\pi n T)-1}\left(1 - \frac{2mM_0^2}{R(4\pi^2 n^2 T^2 m^2 - M_0^4)}\right), \tag{4.4}$$

where $g(x)$ is defined in (4.3). When $m \to \infty$ or $m \to 0$ Eq. (4.4) reduces to (3.12).

It is important to note that the Casimir energy at finite temperature given by the sum (3.12) is free of any divergencies. It is a direct consequence of using, under derivation of (3.12), the integral representation (4.1) for the *renormalized* Casimir energy at zero temperature.

In the limit $T \to \infty$ the Casimir energy (4.4) tends to the value

$$E_C(R, T \to \infty, m) = -\frac{T}{2}\left(1 + \frac{2m}{M_0^2 R}\right) . \tag{4.5}$$

At low temperature the estimation of the sum in (4.4) can be done by making use of the Euler-Maclaurin formula (3.14) again. In order to find the second term in the right-hand side of (3.14) the quantity $F'(0)$ should be calculated, where

$$F(x) = \frac{x(1-\eta x)^2}{e^{2Rx}(1+\eta x)^2 - (1-\eta x)^2}\left(1 - \frac{2\eta}{R(\eta^2 x^2 - 1)}\right) . \tag{4.6}$$

Here η is the ratio m/M_0^2. Expansion of $F(x)$ in series gives

$$F(x) \simeq \frac{1}{2(R+2\eta)}\left[1 - (R+2\eta)x + \mathcal{O}(x^2)\right]\left[1 + \frac{2\eta}{R}(1+\eta^2x^2) + \mathcal{O}(x^4)\right] . \tag{4.7}$$

¿From here it follows that

$$F'(0) = -\frac{1}{2}\left(1 + \frac{2\eta}{R}\right) \to -\frac{1}{2} \quad \text{when} \quad R \to \infty\,. \tag{4.8}$$

In the limit $R \to \infty$ the integral term in the right-hand side of (3.14) vanishes. Hence, the critical temperature in the model under investigation turns out to be the same as that in the Nambu-Goto string with fixed ends

$$\tau_c^2 = \left(\frac{T_c}{M_0}\right)^2 = \frac{3}{\pi(D-2)}\,. \tag{4.9}$$

If we again introduce into consideration the critical temperature dependent on the string length (see the end of the preceding Section) then this quantity proves to be dependent on the quark mass. Indeed, now we have, instead of (3.17),

$$\tau_c^2(\rho,\mu) = \left(1 + \frac{2\mu}{\rho}\right)^{-1}\left(\tau_c^2 - \frac{I(q)}{4\rho^2}\right) , \tag{4.10}$$

where $\mu = m/M_0,\ \rho = M_0 R,\ q = M_0^2 R/m = \rho/\mu$, and the function $I(q)$ generated by the integral term in (3.14) is given by

$$I(q) = \frac{24}{\pi^2}\int\limits_0^\infty \frac{dz\, z(z-q)}{e^{2z}(z+q)^2 - (z-q)^2}\,\frac{z^2 - q^2 - 2q}{z+q}\,. \tag{4.11}$$

When $m \to \infty$ $I(q)$ tends to 1. In order to obtain in this limit Eq. (3.17) from (4.10), one has to direct ρ to ∞ in the first multiplier in (4.10).

5 Conclusion

The method for calculating the Casimir energy at finite temperature proposed here enables us to find the critical temperature in the Nambu-Goto string with massive ends. Beforehand it was not obvious that this temperature proves to be the same as that in the string model with fixed string ends. Probably it is owing to that in string models, like in statistical models, the boundary effects appear to be unessential for implementation of phase transitions.

References

[1] G. Lambiase and V. V. Nesterenko, Phys. Rev. D **54**, 1 (1996).

[2] H. Kleinert, G. Lambiase, and V. V. Nesterenko, Phys. Lett. B (1996).

[3] O. Alvarez, Phys. Rev. D **24**, 440 (1981).

[4] R. D. Pisarski and O. Alvarez, Phys. Rev. D **26**, 3735,(1982).

[5] K. D. Born et al., Phys. Lett. B **329**, 325 (1994).

[6] M. Flesburg and C. Peterson, Nucl. Phys. **B283**, 141 (1987).

[7] I. E. Dzyaloshinskii, E. M. Lifshitz and L. P. Pitaevskii, Advances in Physics, **10**, No. 38, 165 (1961).

[8] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics* (Pergamon, Oxford, 1980), Part 2.

[9] A. Antillón and G. German, Phys. Rev. D **47**, 4567 (1993).